Binary Game # 1

This is a moderately hard game. To solve it, we'll use the following schematic:

Residential = 3 = 3 fire-escapes B / W	Commercial = 4 = 1 fire-escape B / W / S / C
___ ___ ___	___ ___ ___ ___ B

There are 7 buildings. 3 Residential + 4 Commercial = 7 thus all of the buildings are accounted for. Because there are exactly 3 residential buildings and all of these buildings have fire-escapes, and there are a maximum of 4 fire-escapes, we know that only one commercial building has a fire-escape. Thus, only one commercial building is made of brick. However, we do not know how many Residential buildings are made of brick.

1. Which of the following must be true?

   I. At least one of the buildings is made of wood.
   II. At least one of the buildings is made of steel.
   III. At least one of the buildings is made of brick.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) I and III only

Statement I is false. All three residential buildings could be brick and one commercial building could also be brick to give four buildings with fire escapes. This eliminates (A), (D), and (E). Statement II is false. With the same scenario as in Statement I, the three remaining commercial buildings could all be concrete. This eliminates (B). Hence, we have learned that the answer is (C).

2. If exactly two of the commercial buildings are made of concrete, then which one of the following must be true?

   (A) At least one of the commercial buildings is made of wood.
   (B) Exactly two buildings are made of wood.
   (C) Exactly one of the commercial buildings is made of steel.
   (D) No commercial building is made of wood.
   (E) At most one building is made of steel.

Residential = 3 = 3 fire-escapes B / W	Commercial = 4 = 1 fire-escape B / W / S / C
___ ___ ___	___ ___ ___ ___ B C C

If two of the commercial buildings are made of steel and one of the commercial buildings is necessarily brick, then at the most of each material that could be used would be 5 brick (3 residential, 2 commercial),1 steel (1 commercial), 3 wood (3 residential), and 3 concrete (3 commercial). Looking through the answer choices we see the answer is E.

3. If there is at least one wooden building, one concrete building, one brick building, and one steel building, then which one of the following must be false?

(A) Exactly four buildings are made of brick.
(B) Exactly three buildings are made of wood.
(C) Exactly two buildings are made of wood and exactly two buildings are made of steel.
(D) Exactly two buildings are made of steel and exactly two buildings are made of concrete.
(E) Exactly two buildings are made of wood and exactly two buildings are made of brick.

Residential = 3 = 3 fire-escapes B / W	Commercial = 4 = 1 fire-escape B / W / S / C
___ ___ ___	___ ___ ___ ___ B C S

The **answer is D** because if there were an additional building of steel and concrete that would mean there would have to be 5 commercial buildings and the maximum is 4. All of the other answer choices are possible.

4. If there are exactly three brick buildings and one steel building, then any of the following can be true EXCEPT

(A) there is exactly one wooden building
(B) there are no wooden buildings
(C) there are exactly three wooden buildings
(D) there are no concrete buildings
(E) there are exactly two concrete buildings

Residential = 3 = 3 fire-escapes B / W	Commercial = 4 = 1 fire-escape B / W / S / C
___ ___ ___ B B	___ ___ ___ ___ B S

If there are no wooden buildings, then the three residential buildings must all be brick. Now we need a fourth building that has a fire escape. But all the remaining commercial buildings are either steel or concrete, none of which can have a fire escape. Hence the answer is (B).

5. If exactly half of the buildings with fire escapes are wooden, then which one of the following must be false?

(A) There are more wooden buildings than brick buildings.
(B) There are more steel buildings than wooden buildings.
(C) There are exactly three wooden buildings.
(D) There are exactly three brick buildings.
(E) The number of steel buildings is equal to the number of concrete buildings.

Residential = 3 = 3 fire-escapes B / W	Commercial = 4 = 1 fire-escape B / W / S / C
___ ___ ___ W W	___ ___ ___ ___ B

All brick buildings have fire escapes. Now if three of the buildings are brick, choice (D), then there must be three wooden buildings with fire escapes—since half of the buildings with fire escapes are wooden. This gives a total of six buildings with fire escapes. But this violates the condition that exactly four buildings have fire escapes. The answer is (D).

6. If as many as possible of the buildings with fire escapes are wooden, then which of the following must be true?

I. There are exactly three wooden buildings.
II. There is exactly one brick building.

III. There are fewer concrete buildings than wooden buildings.

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

In order to have as many of the wooden buildings with fire escapes as possible, all three residential buildings must be wooden.

3	4
Res	Com
W W W	

Now in order to have four buildings with fire escapes, exactly one of the commercial buildings must be brick:

3	4
Res	Com
W W W	B

Binary Game # 2

The condition "Neither K nor I can be selected with H" can be symbolized as

H—>~K
H—>~I

The condition "Neither L nor G can be selected unless the other is also selected" simply means that if either L or G is selected then both must be selected:

L<—>G

Symbolizing the remaining conditions yields the following schematic:

G or I
H or K
H—>~K
H—>~I
L<—>G

7. Which of the following groups is an acceptable selection of the items?

(A) G, I, L, M
(B) I, K, M, H
(C) G, K, I, M
(D) G, L, J, M
(E) I, G, K, L

Choices (A) and (D) violate the condition H or K. Choice (B) violates the condition H—>~K. Choice (C) violates the condition L<—>G. Hence, by process of elimination, the answer is (E).

8. Which of the following groups of items cannot be among the items selected?

(A) H, J
(B) H, J, M
(C) L, K, I
(D) G, H, M
(E) L, H, J

Begin with (A). Selecting both H and J will satisfy all the conditions, eliminate (A).

Turning to choice (B), since H is selected, I cannot be selected (H—>~I). Hence, from the condition G or I, we must select G. Now, from the condition L<—>G, we must select L. This scenario has five items being selected, violating the fact that only four items are selected. The answer is (B).

9. If I and M are selected, which of the following items must also be selected?
   (A) G, L
   (B) J, H
   (C) H
   (D) K, J
   (E) L

Since I is selected, the condition H—>~I† prevents H from being selected. Hence, the condition H or K forces K to be selected. Now, neither G nor L can be selected since they must be selected together, which would yield a group of five. This leaves only J to be selected. The answer is (D).

10. There would be only one possible way to select the four items if which of the following restrictions were added to the original set of conditions?
    (A) If I is selected, then G is selected.
    (B) Both I and G are selected.
    (C) If J is selected, then M is selected.
    (D) Either L or M is selected.
    (E) If I is selected, then K is selected.

Begin with choice (A). If I is actually selected, then the four items selected would be fully determined. But choice (A) does not require that I be selected. Suppose G is selected.* Then L must be selected since G and L must be selected together. Now, we can satisfy all the conditions by selecting either H and M or H and J. Hence, the items selected are not fully determined, eliminate choice (A). Turning to choice (B), from the condition L<—>G, we know L must be selected. Now, since I has been selected, H cannot be selected (H—>~I). Hence, from the condition H or K, we know K must be selected. Thus, the four items are uniquely determined—I, G, L, K. The answer is choice (B).

Binary Game # 3

The variables are: C C L L L M M

Knowing that there will always be at least two judges voting for each side and one conservative judge will always vote against:

For	Against
___ ___	__C__ ___

The conditional statements can be mapped out as:

If LLL -> ~C
If CCL -> MM

11.

(A) No. This choice has both conservatives and one liberal voting against Datalog. Hence, the hypothesis of Condition 1 is satisfied, and therefore both moderates voted the same way. However, this violates the supplemental condition that *"the two moderates did not vote the same way as each other."*

(B) Yes. This choice has exactly one conservative voting for Datalog. Hence, the other conservative voted against Datalog, which satisfies Condition 4. Again, this choice has exactly one conservative and exactly one liberal voting for Datalog. Hence, the other two liberals voted against Datalog. This gives two people voting for Datalog (one conservative and one liberal) and two people voting against Datalog (two liberals). Hence, Condition 3 is satisfied. Further, Conditions 1 and 2 do not apply. Hence, none of the conditions are violated.

Note, we did not need the supplementary condition *"the two moderates did not vote the same way as each other."* This condition was introduced to make some of the other answer-choices incorrect.

(C) No. This violates Condition 2.

(D) No. Since the moderates split their vote, this violates Condition 1.

(E) No. Since the moderates split their vote, this violates Condition 1.

12.

(A) No. If both conservatives voted against Datalog and all three liberals voted for Datalog, then all the conditions are satisfied.

(B) No. If both conservatives voted against Datalog and all three liberals voted for Datalog, then all the conditions are satisfied.

(C) Yes. Suppose all three liberals voted against Datalog. Then from Condition 2, we know that both conservatives voted for Datalog. However, this violates Condition 4. Hence, at least one liberal voted for Datalog.

(D) No. If both conservatives voted against Datalog and all three liberals voted for Datalog, then all the conditions are satisfied. The moderates can cast their votes in any combination.

(E) No. If both conservatives voted against Datalog and all three liberals voted for Datalog, then all the conditions are satisfied. The moderates can cast their votes in any combination.

13.

(A) No. In Question 14, we learned that at least one liberal voted for Datalog. So if all three liberals voted the same way, then they must have all voted for Datalog. Now, Condition 2 forces both conservatives to vote against Datalog. This already satisfies all the conditions. Hence, one or both moderates could vote against Datalog.

(B) No. In Question 14, we learned that at least one liberal voted for Datalog. So if all three liberals voted the same way, then they must have all voted for Datalog. Now, Condition 2 forces both conservatives to vote against Datalog. This already satisfies

all the conditions. Hence, one or both moderates could vote for Datalog.

(C) No. In Question 14, we learned that at least one liberal voted for Datalog. So if all three liberals voted the same way, then they must have all voted for Datalog. Now, Condition 2 forces both conservatives to vote against Datalog.

(D) No. In Question 14, we learned that at least one liberal voted for Datalog. So if all three liberals voted the same way, then they must have all voted for Datalog. Now, Condition 2 forces both conservatives to vote against Datalog. This already satisfies all the conditions. Hence, both moderates could vote for Datalog or both moderates could vote against Datalog.

(E) Yes. In Question 14, we learned that at least one liberal voted for Datalog. So if all three liberals voted the same way, then they must have all voted for Datalog.

14.

(A) Yes. From Condition 4, we know that at least one conservative voted against Datalog. Suppose one moderate also voted against Datalog—contradicting choice (A). This forces the three liberals to vote for Datalog. So from Condition 2, we know that both conservatives voted against Datalog. However, this violates our assumption that only one conservative and only one moderate voted for Datalog.

(B) No. If exactly two conservatives voted against Datalog and all the other judges voted for Datalog, then all the conditions are satisfied.

(C) No. If exactly one liberal and one conservative voted against Datalog and all the other judges—including one conservative—voted for Datalog, then all the conditions are satisfied.

(D) No. If exactly two conservatives voted against Datalog and all the other judges voted for Datalog, then all the conditions are satisfied.

(E) No. If exactly one liberal and one conservative voted against Datalog and all the other judges voted for Datalog, then all the conditions are satisfied.

15.

(A) No. If both conservatives and both moderates vote against Datalog, then exactly two liberals can vote for Datalog.

(B) No. If exactly two liberals and both moderates vote against Datalog, then exactly one conservative and exactly one liberal must vote for Datalog.

(C) No. If only the conservatives vote against Datalog, then exactly two moderates and exactly three liberals must vote for Datalog.

(D) No. If the only judges to vote against Datalog are one conservative and one liberal, then exactly one conservative, two moderates, and two liberals must vote for Datalog.

(E) Yes. Choice (E) has six of the seven judges voting for Datalog. This violates Condition 3, which states “at least *two* [judges] voted against Datalog.”

16.

(A) No. Condition 4 says at least one conservative voted against Datalog, so if both conservatives voted the same way, both conservatives voted against Datalog.

(B) Yes. Condition 4 says at least one conservative voted against Datalog, so if both conservatives voted the same way, both conservatives voted against Datalog. Additionally, the liberals split their vote, so at least one voted for Datalog and at least one against. Summarizing, the two conservatives and at least one liberal voted against Datalog's petition. Condition 1 then requires that both moderates voted this way. So the moderates cannot vote for Datalog.

(C) No. The liberals split their vote, so at least one voted for Datalog and at least one against.

(D) No. Condition 4 says at least one conservative voted against Datalog, so if both conservatives voted the same way, both conservatives voted against Datalog. Additionally, the liberals split their vote, so at least one voted for Datalog and at least one against. Summarizing, the two conservatives and at least one liberal voted against Datalog's petition. Now, Condition 1 requires that both moderates voted this way. Thus, at least 5 judges voted against Datalog. Further, since "*at least two of the judges voted for Datalog,*" the two remaining judges—liberals—voted for Datalog.

(E) No. Condition 4 says at least one conservative voted against Datalog, so if both conservatives voted the same way, both conservatives voted against Datalog. Additionally, the liberals split their vote, so at least one voted for Datalog and at least one against. Summarizing, the two conservatives and at least one liberal voted against Datalog's petition. Now, Condition 1 requires that both moderates voted this way. Thus, at least 5 judges voted against Datalog. Further, since "*at least two of the judges voted for Datalog,*" no more than 5 judges voted against Datalog. In other words, exactly 5 judges voted against Datalog.