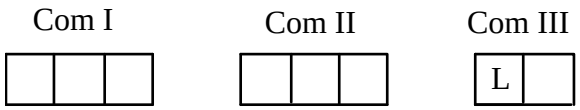
Grouping Game # 1

We start by symbolizing the conditions. The condition *"G serves with M"* is naturally symbolized as **G = M**. The condition *"F does not serve with M"* is symbolized as **F ≠ M**. The condition *"L serves with only one other person"* means that L is on the committee of two; we symbolize it as **L = 2**. The diagram will consist of three compartmentalized boxes. This gives the following schematic:

F G H I J K L M (H, I, J, K "wild")
G = M
F ≠ M
L = 2



Before turning to the questions, two readily derived conditions should be noted. First, since G serves with M, and F does not serve with M, F cannot serve with G. Second, since L serves on the two-person committee, L cannot serve with G or M (otherwise L would be on a three-person committee).

1. Which one of the following is a committee?

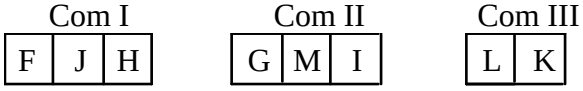
(A) M, L, I
(B) G, F, M
(C) G, L
(D) G, H, I
(E) K, G, M

(A) is not a committee since L must serve on a committee of two. (B) is not a committee since F cannot serve with G. Neither (C) nor (D) is a committee since G and M must serve together. Hence, by process of elimination, the answer is (E).

2. If F cannot serve with K, and K cannot serve with M, which one of the following must be false?

(A) F serves with L.
(B) F serves with J.
(C) L serves with H.
(D) H serves with I.
(E) I serves with M.

We shall use an indirect proof. Start with (A). If F serves with L, then G and M could serve on Committee I, K on Committee II, and the remaining people could serve at random without violating any initial condition. So F could serve with L. This eliminates (A). Next, test (B). If F serves with J on Committee I, then G and M would have to serve on Committee II. And the remaining people could be placed as follows:



This diagram does not violate any initial condition, so F could serve with J. This eliminates (B). Next, test (C). There are two possible places for the pair G and M, Committee I and Committee II. If G and M serve on Committee I, then F would have to serve on Committee II:

Com I			Com II			Com III	
G	M		F			L	H

Clearly, this diagram leaves no room for K since K cannot serve with either M or F. The case with the pair G and M serving on Committee II leads to a similar result. Hence, L cannot serve with H. The answer is (C).

3. If H serves with K, which one of the following cannot be true?
   (A) F serves with K.
   (B) J serves with F.
   (C) I serves with M.
   (D) F serves with L.
   (E) J serves with L.

If H serves with K on Committee I, then G and M must serve on Committee II. (Why?) This gives the following diagram:

Com I			Com II			Com III	
H	K		G	M		L	

(The diagram with H and K on Committee II is not presented because it generates the same results.) Again, we apply an indirect proof. Start with (A). If F serves with K, then from the above diagram F must serve on Committee I. And we can place I and J on Committees II and III, respectively:

Com I			Com II			Com III	
H	K	F	G	M	I	L	J

This diagram does not violate any initial condition, so F could serve with K. This eliminates (A). Next, test (B). J and F cannot serve on Committee I, since from the above diagram H and K are already there. Likewise, J and F cannot serve on Committees II and III. Hence the answer is (B).

4. If K, J, and I serve on different committees, which one of the following must be true?
   (A) K serves with G.
   (B) I serves on a committee of two.
   (C) J serves on a committee of two
   (D) H serves with F.
   (E) J serves with F.

We shall construct counter-examples for four of the answer-choices; the one for which we cannot construct a counter-example will be the answer. Start with (A). Suppose K serves on Committee I and G serves on Committee II. Then from the condition **G = M**, we know that M must also serve on Committee II. And the remaining people can be placed without violating any initial condition as follows:

Com I			Com II			Com III	
K	H	F	J	G	M	L	I

This diagram is a counter-example not only to (A) but to (C) and (E) as well. This eliminates (A), (C), and (E). Next, test choice (B). Suppose that I serves on Committee I, with G and M. Then the remaining people can be grouped as follows:

Com I			Com II			Com III	
I	G	M	F	H	J	L	K

This diagram does not violate any initial condition, so it is a counter-example to (B). Hence, by process of elimination, the answer is (D).

5. Which one of the following conditions is inconsistent with the given conditions?
   (A) K serves on a committee of three.
   (B) M serves with H.

(C) M, H, and I serve together.
(D) F does not serve with G.
(E) H serves with L.

The first counter-example in Question 4 shows that K can serve on a three-person committee. This eliminates (A). Next, turning to choice (B), suppose M serves with H on Committee I. This forces G to also serve on Committee I. Now place F on Committee III and the remaining people as follows:

Com I			Com II			Com III	
M	H	G	J	K	I	L	F

This diagram does not violate any initial condition, so "M serves with H" is consistent with the initial conditions. This eliminates (B). Next, turning to choice (C), suppose M, H, and I serve together on Committee I. But since M must serve with G, there would then be four people on Committee I. The same result occurs when M, H, and G are on Committee II. Hence (C) is inconsistent with the initial conditions, and the answer is (C).

Grouping Game # 2

The diagram for this game can be drawn as follows:

Workers Compensation	Criminal Law	Patent Law
E	D	D

D and F both practice in at least one of the same fields.	DF min 1
D practices in worker's compensation and patent law.	Draw on diagram
F practices in only two fields.	F= 2
D and E do not practice in the same field.	D~E
F and H do not practice in the same field.	F~H

Because each must specialize in at least one of the above fields, but D and E are never together, E must specialize in Workers' Compensation.

We also know that H can only work in one field because F and H do not work together, and F specializes in 2 fields.

6. Which one of the following must be false?

(A) F practices in exactly two fields.
(B) H practices in exactly one field.
(C) E practices in more than one field.
(D) E practices in only one field.
(E) D practices in exactly two fields.

The diagram clearly shows that (A), (B), (D), and (E) are true and that (C) is false. Thus the answer is (C).

7. The people in which one of the following pairs could practice in exactly the same fields?

(A) D and H
(B) E and F
(C) D and E
(D) E and H
(E) H and F

D practices in two fields, whereas H practices in only one. This dismisses (A). Working through the conditions we see that DF must work together at least once but E cannot work with D, thus (B) is eliminated. DE cannot work together, nor can H and F, thus (C) and (E) are eliminated. Thus (D) is correct. You can work through the answer to double check.

8. If the combination of fields in which F practices is different from any of the combinations in which her colleagues practice, then which one of the following must be true?

(A) H does not practice patent law.
(B) F does not practice patent law.
(C) H does not practice worker's compensation.
(D) F practices criminal law.
(E) F and H practice in the same fields.

From the diagram, we know that F must practice either worker's comp. or patent law; but because of the new condition, she cannot practice both—otherwise she would practice in the same fields as D. So F must practice criminal law, and the answer is (D).

9. If a new partner who practices in exactly two fields joins the firm, then he cannot practice in all of the fields that the combination

(A) D and F do
(B) E and H do
(C) E and F do
(D) D and H do
(E) F and H do

Again, from the diagram, we see that F and H practice in mutually exclusive fields. Furthermore, F practices in two fields and H practices in one field, so between them they practice in all three fields. But we are told that the new partner practices in only two fields. Hence, he cannot practice in as many fields as do F and H combined. The answer is (E).

Grouping Game # 3

From the first two conditions we can draw the following diagrams:

Group 1	Group 2	Group 3
___	___ ___	___ ___ ___

Or

Group 1	Group 2	Group 3
___ ___	___ ___	___ ___

Or

Group 1	Group 2	Group 3
____	___	___ ___ ___ ____

The other conditions are:

~Y3 -> ~X3

V~W

10. Which one of the following is an acceptable grouping of the six items?

	Group 1	Group 2	Group 3
(A)	UV	WXY	Z
(B)	X	Y	VZUW
(C)	V	YW	XUZ
(D)	V	Z	XYUW
(E)	UW	YZ	XV

(A) No. This violates the condition "The number of items in Group 2 is less than or equal to the number of items in Group 3."

(B) No. This violates the condition "V and W cannot be in the same group."

(C) No. This violates the condition "X can be in Group 3 only if Y is in Group 3."

(D) Yes. The four conditions are satisfied:

(E) No. This violates the condition "X can be in Group 3 only if Y is in Group 3."

11. If Group 1 contains only the item Y, which of the following must be true?

(A) Group 3 contains four items.

(B) Group 2 contains the same number of items as Group 3.

(C) V is in Group 3.

(D) Group 2 contains three items.

(E) X is in Group 2.

Group 1	Group 2	Group 3
_Y_	___ ___	___ ___ ___

Y is not in 3 thus X cannot be in Group 3, nor can it be in group 1. Thus the answer is (E).

12. If W and Y are in the same group and V is in Group 3, then which of the following must be false?
    (A) W and Y are in Group 2.
    (B) U is the only item in Group 1.
    (C) X is the only item in Group 1.
    (D) U is in Group 3.
    (E) Group 3 contains 2 items.

(A) No. The following grouping satisfies all the conditions and has W and Y in Group 2:

Group 1	Group 2	Group 3
X	WY	UVZ

(B) Yes. Place U and V on a diagram:

Group 1	Group 2	Group 3
U		V

Since "V and W cannot be in the same group," W must be in Group 2. Further, since "W and Y are in the same group," Y must also be in Group 2:

Group 1	Group 2	Group 3
U	WY	V

Now, since "the number of items in Group 2 is less than or equal to the number of items in Group 3," the remaining items must be in Group 3:

Group 1	Group 2	Group 3
U	WY	VXZ

However, this diagram violates the condition "X can be in Group 3 only if Y is in Group 3."

(C) No. The following grouping satisfies all the conditions and has X as the only item in Group 1:

Group 1	Group 2	Group 3
X	WY	UVZ

(D) No. The following grouping satisfies all the conditions and has U in Group 3:

Group 1	Group 2	Group 3
WY	XZ	VU

(E) No. The following grouping satisfies all the conditions and has 2 items in Group 3:

Group 1	Group 2	Group 3
WY	XZ	VU

Thus the answer is B.

13. If Group 2 contains only one item which is neither W nor V, which of the following must be true?
    (A) Group 1 contains only V or only W.
    (B) Group 3 contains W.
    (C) Group 1 contains both U and V.
    (D) Group 2 contains Z.
    (E) Group 1 contains only W.

Since Group 2 contains only one item, Group 1 must contain only one item: "The number of items in Group 1 is less than or equal to the number of items in Group 2." Now, Group 3 cannot contain both V and W since "V and W cannot be in the same group." Hence, either V or W must be in Group 1 as neither can be in Group 2. The answer is (A).

Grouping Game # 4

This problem is rather convoluted because not only are there direct conditions on the players, such as *"Drexler and Bird do not both start,"* but there are also constraining numerical conditions, such as *"exactly three fast-break specialists must be chosen."*

It is best to solve this problem without a diagram; however, we will still symbolize the conditions for clarity and easy reference. The condition *"Jordan starts only if Bird starts"* implies only that if Jordan is starting then Bird must be starting as well. So we symbolize it as **Jordan—>Bird**. The condition *"Drexler and Bird do not both start"* means that if one starts then the other does not. So we symbolize it as **Drexler—>~Bird**.* Students often misinterpret this condition to mean that neither of them starts. To state that neither starts, put *both* at the beginning of the sentence: *Both* Drexler and Bird do not start.

The condition *"if Jordan starts, then Malone does not"* is naturally symbolized as **Jordan—>~Malone**. It tells us that if J starts then M does not, but tells us nothing when M does not start. Such a condition, where the two parts of an *if-then* statement do not similarly affect each other, is called a nonreciprocal condition. On the other hand, a condition such as Jordan<—>~Malone affects J and M equally. In this case, we are told that if J starts then M does not as before, but we are told additionally that if M does not start then J does. It is important to keep the distinction between reciprocal and nonreciprocal relations clear; a common mistake is to interpret a nonreciprocal relation as reciprocal. The remaining conditions cannot be easily written in symbol form, but we will paraphrase them in the schematic:

Jordan—>Bird
Drexler—>~Bird
Jordan—>~Malone
2 from Group A 3 from Group B
fast-break specialists: Johnson, Bird, Jordan, Pippen
3 fast-break specialists
Ewing, Laettner, Robinson are "wild"

Note: Ewing, Laettner, and Robinson are independent because there are no conditions that refer directly to them. We now turn to the questions.

* **Bird—>~Drexler** would also suffice, but we don't need both.

14. If Jordan starts, which of the following must also start?

    (A) Malone or Johnson
    (B) Drexler or Laettner
    (C) Drexler or Johnson
    (D) Johnson or Pippen
    (E) Malone or Robinson

From the condition **Jordan—>Bird**, we know that if Jordan starts, then Bird must start as well. Now both Jordan and Bird are fast-break specialists, and three of the four fast-break specialists must start. So at least one of the remaining fast-break specialists—Johnson or Pippen—must also start. The answer is (D).

15. All of the following pairs of players can start together EXCEPT:

    (A) Ewing and Drexler
    (B) Jordan and Johnson
    (C) Robinson and Johnson
    (D) Johnson and Bird
    (E) Pippen and Malone

We shall use the method of indirect proof to solve this problem: That is, assume that a particular answer-choice is true. Then check whether it leads to a contradiction or an impossible situation. If so, it is the answer; if not, then select another answer-choice and repeat the process until a contradiction is found.

Begin with choice (A). Both Ewing and Drexler are from Group A, so the remaining 3 starters must be chosen from Group B. Additionally, they must all be fast-break specialists since neither E nor D is—there are exactly 3 fast-break specialists. But Jordan and Pippen are the only fast-break specialists in Group B. So the third fast-break specialist cannot be chosen. The answer therefore is (A). This type of question can be time consuming because you may have to check all the answer-choices—save these questions for last.

16. If the condition "Bird starts only if Pippen doesn't" is added to the other conditions, then which one of the following must be false?

    (A) Johnson starts with Bird
    (B) Laettner starts with Malone
    (C) Laettner starts with Bird
    (D) Jordan starts with Robinson
    (E) Jordan starts with Bird

This problem is both long and hard. Again, we use an indirect proof. Start with (A). Both Johnson and Bird are from Group A, and both are fast-break specialists. So the remaining 3 starters must be chosen from Group B, one of which must be a fast-break specialist. Now if Jordan, Robinson, and Laettner are chosen, there will be three fast-break specialists and none of the initial conditions will be violated. So (A) is not necessarily false; eliminate it. Next, we check (B). Both Laettner and Malone are from Group B, and neither is a fast-break specialist. So the three remaining starters must all be fast-break specialists, and two of them must be from Group A—Johnson and Bird. This leaves only Jordan and Pippen to choose from. Jordan cannot be chosen because Malone has already been chosen (**Jordan—>~Malone**), and from the new condition Pippen cannot be chosen because Bird has already been chosen. Hence the answer is (B).

17. If Malone starts, which one of the following is a complete and accurate list of the players from Group A any one of whom could also start?

    (A) Johnson
    (B) Johnson, Drexler
    (C) Johnson, Bird

(D) Johnson, Drexler, Bird
(E) Johnson, Ewing, Bird

Jordan *cannot* start with Malone according to the condition **Jordan—>~Malone**. To play three fast-break specialists, therefore, Johnson, Bird, and Pippen are all *required* to start. Since both Johnson and Bird are from Group A and exactly two players from that group start, these two players comprise the complete list of starters from Group A when Malone also starts. The answer is (C).

18. Which one of the following players must start?

(A) Pippen
(B) Johnson
(C) Jordan
(D) Malone
(E) Bird

Suppose Bird does not start. Then the 3 fast-break specialists must be Johnson, Jordan, and Pippen. But if Jordan starts, then from the initial conditions Bird must also start. Hence Bird must always start. The answer is (E).