Solutions to Chm220 Practice Exam

Q1)

Q1)

a) for 1-step process: $W = +1.95\ kJ$

$\downarrow\downarrow\downarrow\ P_{ext} = 5.5 atm$

$$\begin{array}{|c|}\hline 0.2 mol \\ V_i \\ 1 atm \\ 298K \\ \hline\end{array} \xrightarrow[w>0]{isothermal} \begin{array}{|c|}\hline 0.2 mol \\ V_f \\ P_f \\ 298K \\ \hline\end{array}$$

$$V_i = \frac{nRT}{P_i} = \frac{(0.2 \cancel{mol})(8.314 \frac{J}{\cancel{mol}\,\cancel{K}})(298\cancel{K})}{101325\ Pa}$$

$= 0.005\ m^3$

$= 5\ L$

$P_{ext} = 5.5 atm$

$$W = -P_{ext}\,\Delta V = -P_{ext}(V_f - V_i)$$

$$1{,}950\,J = -5.5\,\cancel{atm} \times 101325 \frac{Pa}{\cancel{atm}}\,(V_f - 0.005 m^3)$$

$-0.0035 m^3 = V_f - 0.005 m^3$

$0.0015\ m^3 = V_f$

$\boxed{1.5\,L = V_f}$

b) for 2-step process: 2 compressions A & B

$\downarrow\downarrow\downarrow\ P_{ext\,A} = \frac{1}{2}(5.5 atm)$ $\qquad \downarrow\downarrow\downarrow\ P_{ext\,B} = 5.5 atm$

$$\underset{①}{\begin{array}{|c|}\hline 0.2 mol \\ V_1 = V_i \\ 1 atm \\ 298K \\ \hline\end{array}} \xrightarrow[W_A>0]{Ⓐ\ isothermal} \underset{②}{\begin{array}{|c|}\hline 0.2 mol \\ V_2 \\ P_2 \\ 298K \\ \hline\end{array}} \xrightarrow[W_B>0]{Ⓑ\ isothermal} \underset{③}{\begin{array}{|c|}\hline 0.2 mol \\ V_3 = V_f \\ P_3 \\ 298K \\ \hline\end{array}}$$

$V_2 = V_i - \frac{1}{2}(V_i - V_f)$

$V_2 = 5L - \frac{1}{2}(5L - 1.5L)$

$V_2 = 3.25\,L$

$$W_A = -P_{ext\,A}(V_2 - V_1)$$

$$= -\frac{1}{2}\left(5.5\,\cancel{atm} \times 101325 \frac{Pa}{\cancel{atm}}\right)(3.25 - 5)\cancel{L} \times \frac{1 m^3}{1000 \cancel{L}}$$

$= +488\ J$

$$W_B = -P_{ext\,B}(V_3 - V_2)$$

$$= -5.5\,\cancel{atm} \times 101325 \frac{Pa}{\cancel{atm}}(1.5 - 3.25)\cancel{L} \times \frac{1 m^3}{1000\cancel{L}}$$

$= +975\ J$

$W_{total} = W_A + W_B = \boxed{1.463\,kJ}$

c) minimum work : ∞ # of small steps
(notice Work decreased from 1.95 kJ to 1.463 kJ by changing from a 1-step to a 2-step process)

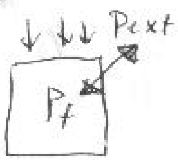
→ Change Pext to be infintessimely larger than the P in the piston/cylinder

→ = a reversible process

$$W_{rev} = -nRT \ln \frac{V_f}{V_i}$$

$$= -(0.2\,\cancel{mol})(8.314\,\frac{J}{\cancel{mol \cdot K}})(298\,\cancel{K}) \ln\left(\frac{1.5\,\cancel{L}}{5\,\cancel{L}}\right) = \boxed{597\,J}$$

d) minimum Pext for a 1-step process:
- must be equal to the final pressure



pressure equilibrium

- if Pext < P_f, then P_f cannot be reached (piston would move in opposite direction

$$P_{ext,minimum} = P_f = \frac{nRT}{V_f}$$

$$= \frac{(0.2\,\cancel{mol})(8.314\,\frac{\cancel{J}}{\cancel{mol \cdot K}})(298\,\cancel{K})}{0.0015\,\cancel{m^3}} \quad (Pa)$$

$$= 330{,}343\,Pa \times \frac{1\,atm}{101{,}325\,Pa} = \boxed{3.26\,atm}$$

Q2)

a) 35% has reacted

$$[A]_0 \longrightarrow [A] = 0.65[A]_0$$

$t = 0$ $\qquad$ $t = 800\,s$

for a 1st order reaction: $[A] = [A]_0 e^{-kt}$

$$0.65\cancel{[A]_0} = \cancel{[A]_0} e^{-k(800s)}$$

$$\ln(0.65) = -k(800s)$$

$$k = \frac{-\ln(0.65)}{800s} = \frac{5.38\times10^{-4}}{s}$$

$$t_{1/2} = \frac{1}{k} = \frac{1}{\frac{5.38\times10^{-4}}{s}} = \boxed{1857s}$$

b) $k = Ae^{-Ea/RT}$

$$k \text{ without catalyst} = \frac{5.38\times10^{-4}}{s} = A\exp\left(\frac{-129{,}000\frac{\cancel{J}}{\cancel{mol}}}{8.314\frac{\cancel{J}}{\cancel{mol\cdot K}}\times 350\cancel{K}}\right)$$

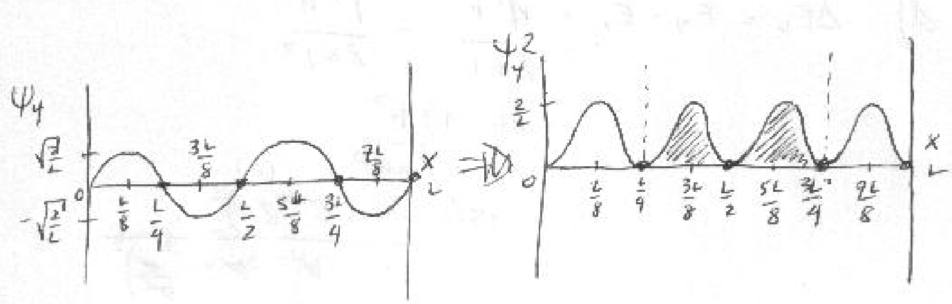
$$A = \frac{9.64\times10^{15}}{s}$$

$$k \text{ with catalyst} = \left(\frac{9.64\times10^{15}}{s}\right)\exp\left(\frac{-87{,}000\frac{\cancel{J}}{\cancel{mol}}}{8.314\frac{\cancel{J}}{\cancel{mol\cdot K}}\times 298\cancel{K}}\right)$$

$$= \boxed{5.42\ s^{-1}}$$

Q3)

a)



b) middle half : $\frac{L}{4}$ to $\frac{3L}{4}$

→ encompases 2 full regions (of the 4) on the ψ_4^2 vs x graph

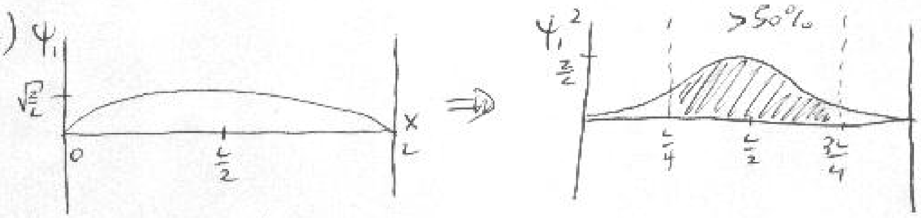
→ each region is equal in size, and the total area is equal to 1 (ψ_4^2 is normalized)

$\therefore P = \int_{L/4}^{3L/4} \psi_4^2(x)\,dx$ = area under the curve of ψ_4^2 vs. x between $L/4$ and $3L/4$

= 0.5

$\boxed{= 50\%}$

c)



$$\int_{L/4}^{3L/4} \psi_1^2(x)\,dx > \int_{L/4}^{3L/4} \psi_4^2(x)\,dx$$

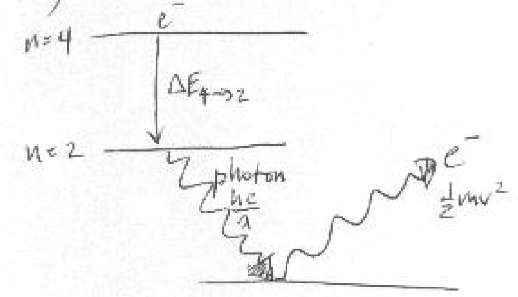
d) $$\Delta E_k = E_4 - E_1 = \frac{4^2 h^2}{8mL^2} - \frac{1^2 h^2}{8mL^2}$$

$$= \frac{(16-1)\,h^2}{8mL^2} \quad (J)$$

$$= \frac{15\left(6.636\times10^{-34}\,J\cdot\cancel{s}\right)^2}{8\left(9.11\times10^{-31}\,\cancel{kg}\right)\left(350\,\cancel{pm}\times10^{-12}\,\frac{m}{\cancel{pm}}\right)^2}$$

$$= \boxed{7.40\times10^{-18}\ J}$$

Q4)



Aluminum

- find λ
- find E_k & v of e^- (if $\frac{hc}{\lambda} > W$)

$$W = 4.08\,\cancel{eV}\left(\frac{1.602\times10^{-19}\,J}{\cancel{eV}}\right) = 6.54\times10^{-19}\,J$$

$$\Delta E_{4\to2} = E_2 - E_4 = -R_H Z^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{-hc}{\lambda}$$

$$\lambda = \frac{+hc}{R_H Z^2\left(\frac{1}{4} - \frac{1}{16}\right)}$$

$$= \frac{+(6.636\times10^{-34}\,\cancel{J}\cdot\cancel{s})(3.0\times10^{8}\,\frac{m}{\cancel{s}})}{(2.18\times10^{-18}\,\cancel{J})(3)^2\left(\frac{1}{4} - \frac{1}{16}\right)}$$

$$= 5.41\times10^{-8}\,\cancel{m}\times10^{9}\,\frac{nm}{\cancel{m}}$$

$$= \boxed{54.1\ nm}$$

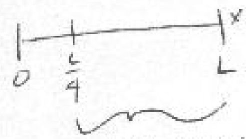
$$\frac{hc}{\lambda} = \frac{(6.636\times10^{-34}\,J\cdot\cancel{s})(3.0\times10^{8}\,\frac{\cancel{m}}{\cancel{s}})}{5.41\times10^{-8}\,\cancel{m}} = 3.68\times10^{-18}\,J$$

$\therefore \frac{hc}{\lambda} > W$ - meaning the emitted photon has more energy than the work function for Al, the threshhold for emitting an e^-

$$\frac{hc}{\lambda} = W + E_k \quad \therefore E_k = 3.68\times10^{-18}\,J - 6.54\times10^{-19}\,J = \boxed{3.02\times10^{-18}\,J}$$

$$E_k = \frac{1}{2}mv^2 \quad \therefore v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(3.02\times10^{-18}\,\cancel{J})\frac{m^2}{s^2}}{9.11\times10^{-31}\,kg}} = \boxed{2.58\times10^{6}\,\frac{m}{s}}$$

Q5)



$$\Psi^2 = \frac{30}{L^5}\left(L^2x^2 - 2Lx^3 + x^4\right)$$

we want the probability in this region ($\frac{L}{4}$ to L)

— but since the total probability (from 0 to L) is $= 1$,

then we can calculate for 0 to $\frac{L}{4}$ instead:

$$\cancel{\int_0^L \Psi^2 dx}^{\,=1} = \int_0^{\frac{L}{4}} \Psi^2 dx + \int_{\frac{L}{4}}^{L} \Psi^2 dx$$

$$\int_{\frac{L}{4}}^{L} \Psi^2 dx = 1 - \int_0^{\frac{L}{4}} \Psi^2 dx$$

$$= 1 - \frac{30}{L^5}\left[\frac{L^2x^3}{3} - \frac{\cancel{2}Lx^4}{{}^{2}\cancel{4}} + \frac{x^5}{5}\right]_0^{L/4}$$

$$= 1 - \frac{30}{L^5}\left[\frac{L^2\left(\frac{L}{4}\right)^3}{3} - \frac{L\left(\frac{L}{4}\right)^4}{2} + \frac{\left(\frac{L}{4}\right)^5}{5}\right]$$

$$= 1 - \frac{30}{\cancel{L^5}}\left[\frac{\cancel{L^5}^{\,1}}{3\times4^3} - \frac{\cancel{L^5}^{\,1}}{2\times4^4} + \frac{\cancel{L^5}^{\,1}}{5\times4^5}\right] \leftarrow \text{common denom} = 30\times4^5$$

$$= 1 - \cancel{30}\left[\frac{10\times4^2}{\cancel{10\times3}\times4^5} - \frac{15\times4}{\cancel{15\times2}\times4^5} + \frac{6}{\cancel{6\times5}\times4^5}\right]$$

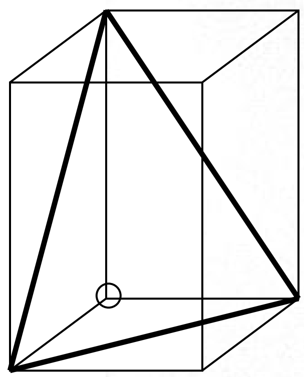
$$= 1 - \left[\frac{160 - 60 + 6}{4^5}\right]$$

$$= \frac{4^5}{4^5} - \frac{106}{4^5} = \boxed{89.6\%}$$

Q6)

b) $\frac{1}{d_{hkl}} = \sqrt{\frac{h^2}{a^2} + \frac{k^2}{a^2} + \frac{l^2}{b^2}}$ for this crystal structure

$$d_{111} = \frac{1}{\sqrt{\frac{1^2}{(310pm)^2} + \frac{1^2}{(310pm)^2} + \frac{1^2}{(450pm)^2}}} = 197\,pm$$



$\lambda = 0.856\,\text{Å}$

$n = 1$

$$\sin\theta = \frac{n\lambda}{2d_{hkl}} = \frac{0.856\times10^{-10}\,\cancel{m}}{2(197\times10^{-12}\,\cancel{m})} = 0.217$$

$$\theta = \boxed{12.54°}$$

c) $$\text{concentration} = \frac{\rho}{MW} = \frac{12.5\,\frac{g}{cm^3}}{81.5\,\frac{g}{mol}} = 0.1534\,\frac{\cancel{mol}}{cm^3} \times 6.022\times10^{23}\,\frac{atoms}{\cancel{mol}}$$

$$= 9.236\times10^{22}\,\frac{atoms}{cm^3}$$

for this crystal structure: $\frac{\text{volume}}{\text{unit cell}} = a^2b = (310\times10^{-12}\,m)^2(450\times10^{-12}\,m)$

$$= 4.32\times10^{-29}\,\frac{\cancel{m^3}}{unitcell} \times \frac{(100\,cm)^3}{\cancel{m^3}}$$

$$= 4.32\times10^{-23}\,\frac{cm^3}{unitcell}$$

$$\therefore \frac{\#\,atoms}{unit\ cell} = \left(9.236\times10^{22}\,\frac{atoms}{\cancel{cm^3}}\right)\left(4.32\times10^{-23}\,\frac{\cancel{cm^3}}{unitcell}\right) = \boxed{4}$$

Q7)

$$\nu_0 = \frac{c}{\lambda_0} \quad \text{where} \quad \lambda_0 = \frac{1}{\text{wavenumber}} = \frac{1\,cm}{3550} \times \frac{1\,m}{100\,cm}$$

$$= 2.82 \times 10^{-6}\,m$$

$$\nu_0 = \frac{3.0 \times 10^8 \,\cancel{m}/s}{2.82 \times 10^{-6}\,\cancel{m}} = 1.065 \times 10^{14}\,s^{-1}$$

$$E_0 = \frac{1}{2} h \nu_0 = \frac{1}{2}(6.63 \times 10^{-34}\,J\cancel{s})(1.065 \times 10^{14}\,\cancel{s^{-1}}) = \boxed{3.53 \times 10^{-20}\,J}$$

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \Longrightarrow k = (2\pi\nu_0)^2 \mu$$

$$\text{where } \mu = \frac{m_F m_H}{m_F + m_H} = \frac{(19)(1)}{19+1}(1.67 \times 10^{-27}\,kg)$$

$$= 1.587 \times 10^{-27}\,kg$$

$$k = \left((2\pi)(1.065 \times 10^{14}\,s^{-1})\right)^2 (1.587 \times 10^{-27}\,kg)$$

$$= \boxed{710 \,\frac{kg}{s^2}}$$