PREP101 Your Competitive Edge

MIE 100 Equation Sheet

General Approach to Solving Problems

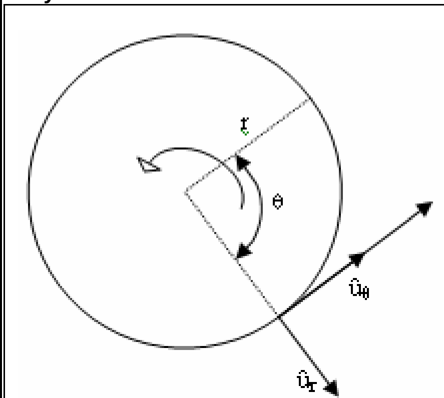
1. Draw a picture. Label known values and unknown variables.
2. Define a coordinate system most natural to the problem: rectilinear, normal-tangential, or cylindrical. Normal-tangential and cylindrical coordinates lend themselves well to circular and curvy motions.
3. Decide on a strategy and find relationships useful to the problem: this is where equations come in. Use kinematic equations of motion, forces, energy conservation, momentum conservation, or a combination of these. Often more than one strategy can be applied to the same problem.
4. Convert all numbers to SI units: mass in kg, length in m, time in s, and substitute into relationships set-up in 3 to find the final answer.

KINEMATIC EQUATIONS OF MOTION

Rectilinear Coordinate

Velocity $v=\frac{ds}{dt}$	*s* = position, *t* = time
Acceleration $a=\frac{dv}{dt}$	*v* = velocity, *t* = time
1-D Linear Motion (constant acceleration *a*) $v(t)=v_0+at$ $s(t)=s_0+v_0t+\frac{1}{2}at^2$ $v^2=v_0^2+2a(s-s_0)$ $a\,ds=v\,dv$	To apply in two dimensions, the easiest way is to choose an x-y coordinate system so that the direction of the acceleration is entirely along either the x or the y direction. This greatly simplifies things as the acceleration in the other coordinate direction will have a component of 0 and the motion in that other direction will have constant velocity. The components of motion in the x and y directions are analyzed separately.
Vector components $v_x=v\cos\theta$ $v=\sqrt{v_x^2+v_y^2}$ $v_y=v\sin\theta$	For a vector of magnitude *v* making an angle θ with the x-axis
$\tan\theta=\frac{v_y}{v_x}$	

Cylindrical / Polar Coordinate



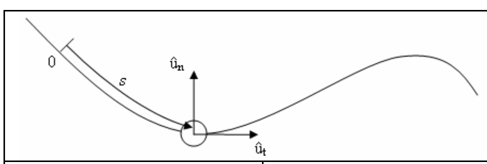
Angular Velocity $\omega=\frac{d\theta}{dt}=\frac{v}{r}$	θ = angle in rads, *r* = radius of curvature
Angular Acceleration $\alpha=\frac{d\omega}{dt}$	ω = angular velocity, *t* = time
Circular motion (constant acceleration *α*) $\omega(t)=\omega_0+\alpha t$ $\theta(t)=\theta_0+\omega_0 t+\frac{1}{2}\alpha t^2$ $a\,d\theta=\omega\,d\omega$	Notice the similarity of these equations to the equations of 1D linear motion. In general, linear motion can find rotational analogues by replacing displacement *s* with angle *θ*, velocity *v* with angular velocity *ω*, acceleration *a* with angular velocity *α*, force *F* with torque *τ*, and mass *m* with moment of inertia *I*.

In vector form

$$\vec v=\dot r\vec u_r+r\dot\theta\vec u_\theta=\frac{dr}{dt}\vec u_r+r\frac{d\theta}{dt}\vec u_\theta$$

$$\vec a=(\ddot r-r\dot\theta^2)\vec u_r+(r\ddot\theta+2\dot r\dot\theta)\vec u_\theta=\left(\frac{d^2r}{dt^2}-r\frac{d\theta^2}{dt}\right)\vec u_r+\left(r\frac{d^2\theta}{dt^2}+2\frac{dr}{dt}\frac{d\theta}{dt}\right)\vec u_\theta$$

Normal-Tangential Coordinate



Tangential Acceleration $a_t=\frac{dv}{dt}=r\frac{d\omega}{dt}=r\alpha$	Acceleration in the same direction as the instantaneous particle trajectory.
Normal Acceleration $a_n=a_c=\frac{v^2}{r}=r\omega^2$	Normal, or centripetal acceleration a_n is toward the center of the circle of radius *r* for an object traveling with constant speed *v*

KINETICS

Forces and Torques

Centre of Mass $x_{cm}=\frac{\sum_{i=1}^{n}m_ix_i}{M_{total}}$	The center of mass is a point that represents the average location for the total mass of the system.
Mass Moment of Inertia $I=\int r^2dm$	The mass moment of inertia is the rotational analogue of mass. It resists rotational acceleration rather than linear acceleration.
Parallel-Axis Theorem $I=I_G+md^2$	I_G is the moment of inertia of an object about an axis through its own centre of mass, which is the minimum moment of inertia about any axis in that direction. To find the moment of inertia about another axis parallel to the centre-of-mass axis, the Parallel-Axis Theorem may be used. m = mass of object d = distance to centre-of-mass axis
Newton's Second Law of Motion (Dynamics) $\vec F=m\vec a=\frac{dp}{dt}$	The acceleration *a* of an object is directly proportional to the net force acting on it and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force action the object. p = momentum
Force of static friction $F_{fr}\le\mu_sF_N$	Opposes any impending relative motion between two surfaces, where the magnitude can assume any value up to a maximum of μ_sF_N where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force.
Force of kinetic friction $F_{fr}=\mu_kF_N$	Force between two surfaces sliding against one another that opposes the relative motion of the two surfaces, where μ_k is the coefficient of kinetic friction.

Force of gravity between any two objects $F_G = G\frac{m_1 m_2}{r^2}$	The force F_G between two objects of masses m_1 and m_2 and separated by a distance *r*. The value of the universal gravitation constant is: $G = 6.67\times10^{-11}$ $N\cdot m^2/kg^2$
Hooke's Law $F = -k\Delta x$	The further a spring is stretched, the more force it pulls back with.
Torque $\tau = r\times F = \frac{dH}{dt} = I\alpha$	Torque, which can be roughly thought of as a twisting force, is proportional to the force applied and the lever arm length. H = angular momentum.

WORK AND ENERGY

Work $U = Fs\cos\theta$	Work *U* done by a constant force of magnitude *F* on an object as it is displaced by a distance *s*. The angle between the directions of *F* and *s* is θ. Work is positive if the object is displaced in the direction of the force and negative if it is displaced against the force. The work is zero if the displacement is perpendicular to the direction of the force.
For Circular Motion $U = \tau\cdot\theta$	For circular motion, τ is the torque acting on the body and θ is the angle by which the body has rotated through the action of τ.
Kinetic Energy $T = \frac{1}{2}mv^2$	Kinetic energy T for a mass *m* traveling at a speed *v*.
For Circular Motion $T = \frac{1}{2}I\omega^2$	Kinetic energy T for an object with moment of inertia I rotating at angular velocity ω.
Gravitational Potential Energy $V = mgh$ (local) $V = -\frac{GMm}{r}$ (general)	Potential energy *V* is the energy that an object of mass *m* has by virtue of its position relative to the surface of the earth. That position is measured by the height *h* of the object relative to an arbitrary zero level.

Spring Potential Energy $V = \frac{1}{2}K\Delta x^2$	The potential energy stored in a spring that is being compressed or stretched.
Conservation of Mechanical Energy (Only holds true if non-conservative forces are ignored) $E_2 = E_1$ $T_2 + V_2 = T_1 + V_1$	The total mechanical energy of a system, remains constant as the object moves, provided that the net work done by external non-conservative forces (such as friction and air resistance) is zero.
Power $P = \frac{dU}{dt} = Fv\cos\theta$	Power *P* is defined as the rate at which work is done. It can also be expressed in terms of the force *F* being applied to the object traveling at a speed *v*.

MOMENTUM

Linear Momentum: $\vec{p} = m\vec{v}$	Linear momentum *p* is the product of an object's mass *m* and velocity *v*. Linear momentum is a vector that points in the same direction of the velocity.
Angular Momentum $H_G = I\omega = r\times p$	The angular momentum vector points out of the plane of rotation according to the right hand rule.
Conservation of Momentum: $\vec{P}_2 = \vec{P}_1$ $H_{G1} = H_{G2}$	The total linear and angular momentum of an isolated system remains constant.
Impulse-momentum theorem: $\Delta p = F\cdot t$ $\vec{p}_2 - \vec{p}_1 = \vec{\bar{F}}_{net}\Delta t$	An impulse produces a change in an object's momentum. Impulse is given by the product of average force $\vec{\bar{F}}$ (*F*) and the time interval Δt (*t*) over which the force is applied. Impulse is a vector that points in the same direction as the average force.

WAVES AND PERIODIC MOTION

Simple Harmonic Motion $\ddot{x} + \omega_n^2 x = 0$ $\omega_n = \sqrt{\frac{k}{m}}$	Differential equation describing displacement of an object on a spring with respect to time. The solution is $x(t) = A\cos(\omega_n t + \phi)$ ω_n = natural frequency ϕ = phase A = amplitude
Vibration Period $T = \frac{1}{f} = \frac{2\pi}{\omega_n}$	
Forced Vibration $x_p = \frac{F_o/k}{1-(\omega_o/\omega_n)^2}\sin\omega_o$	System subjected to periodic force F = $F_o \sin \omega_o t$. k = spring constant
Critical Damping Coefficient $c_c = 2m\omega_n$	Important in damped vibration. When system is damped by force $F = c\dot{x}$, c determines the behaviour. $c > c_c$ overdamped system, returns to equilibrium without vibration $c = c_c$ critically damped system, smallest c value necessary for the system to be non-vibrating $c < c_c$ underdamped system, amplitude of vibration decreases exponentially over time